35. System Classification & Impulse Response

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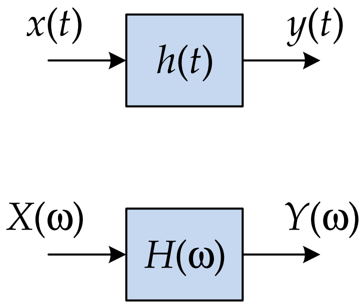
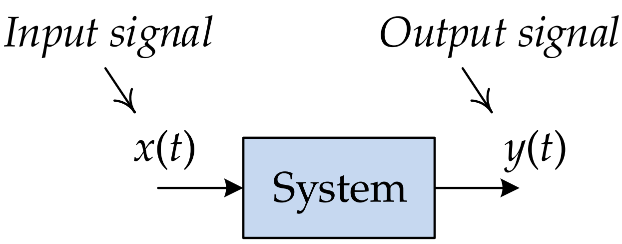


Systems and Signals

A **system** is a physical process, hardware device, or software algorithm that takes an **input** (or excitation) **signal**, acts on it and produces the **output** (or response) **signal**. Multiple input and/or output signals are also possible.



We design and use many systems in engineering built using resistors, capacitors, diodes, transistors, microcontrollers, etc, and/or software.

Linear vs. Nonlinear Systems

Linear system: Satisfies both the additivity property and homogeneity (or scaling) property.

Satisfying **additivity** means:

If we feed input $x_1(t)$ alone to the system, and it produces output $y_1(t)$, and if we feed input $x_2(t)$ alone to the system, it produces output $y_2(t)$, then if we feed $x(t) = x_1(t) + x_2(t)$ to the system, it will necessarily produce the output $y(t) = y_1(t) + y_2(t)$, for any signals $x_1(t)$ and $x_2(t)$.

Satisfying **homogeneity** (scaling) means:

If we feed input $x(t)$ to the system, and it produces output $y(t)$, then if we feed $a\,x(t)$ to the system, it will necessarily produce the output $a\,y(t)$. This applies for any signal $x(t)$ and scalar a.

Both additivity and homogeneity can be combined into one property, called linearity (or superposition), expressed as

$$a\,x_1(t) + b\,x_2(t) \longrightarrow a\,y_1(t) + b\,y_2(t)$$

for any scalars a and b.

A system that does not satisfy additivity and/or homogeneity is a nonlinear system.

Remember that in electric circuits consisting of resistors, capacitors and inductors you managed to use superposition to calculate voltages and currents since those hardware components are linear. For other nonlinear components (e.g., a diode), you are not allowed to use superposition (unless you work in a narrow region that is almost linear).

Time-Invariant vs Time-Varying Systems

Time-Invariant system: A time shift (delay or advance) in the input signal causes the same time shift in the output signal. So, if

$$x(t) \longrightarrow y(t)$$

Then, for any real value of t_0,

$$x(t - t_0) \longrightarrow y(t - t_0)$$

A system which does not satisfy this property is called a **time-varying system**.

A system that is linear and also time-invariant is called LTI system (short for linear time-invariant).

Causal vs. Noncausal Systems

Causal (or physical) system: The output of the system at the present time depends only on the present and/or past values of the input.

Thus, in a causal system, it is not possible to obtain an output before an input is applied to the system.

Noncausal (or anticipative) system: The output of the system at the present time depends on future values of the input.

Example of a noncausal system:

$$y(t) = 5\,x(t+1)$$

Instantaneous (Memoryless) vs. Dynamic (with Memory) Systems

Memoryless (or Instantaneous) system: The output of the system at any time instant depends only on the input(s) at that same time instant.

In other words, past history is irrelevant in determining the response at current time. Memoryless systems are causal.

With Memory (Dynamic) system: The output of the system depends on some past values of the input. If the system response at instant t is dependent on the input signals over the past T seconds (i.e., input(s) within time interval $(t-T, t)$) it is called a finite-memory system with a memory of T seconds.

Memoryless system example: Electric circuit consisting of resistors. Consider one resistor, the output (e.g., voltage across the resistor) is related to the input (e.g., current through the resistor) by Ohm's law:

$$v(t) = R\, i(t)$$

System with memory example: Electric circuit containing capacitors and/or inductors generally have infinite memory. Consider one capacitor, the voltage is related to the current by [notice the dependence on the current values over the entire past history $(-\infty, t)$]:

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau$$

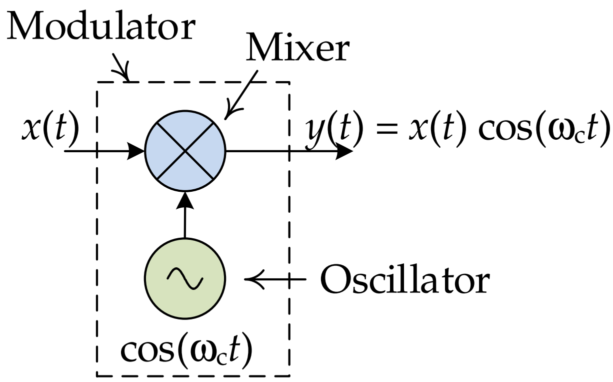
Stable vs. Unstable Systems

Stable system: Every bounded input applied at the input terminal (e.g., $|x(t)| < k_1$) should result in a bounded output (e.g., $|y(t)| < k_2$), where k_1 and k_2 are finite real constants. This system is called BIBO (bounded-input/bounded-output) stable.

Unstable system: Not all bounded inputs lead to bounded output.

For example, the system where the output is $y[n] = (n + 1)\, x[n]$ is not stable, because if the input $x[n] = 1$, which is bounded, the output $y[n]$ can increase without bound as n increases.

Q1. The following analog modulator system has an input signal $x(t)$ and an output signal $y(t) = x(t)\cos(\omega_c t)$. Classify this system, while providing clear explanation.



Linear: If $x(t) = a\,x_1(t) + b\,x_2(t)$, then $y(t) = x(t)\cos(\omega_c t) = [a\,x_1(t) + b\,x_2(t)]\cos(\omega_c t) = a\,x_1(t)\cos(\omega_c t) + b\,x_2(t)\cos(\omega_c t) = a\,y_1(t) + b\,y_2(t)$, which is the superposition of the two outputs in response to $x_1(t)$ and $x_2(t)$ individually.

Time-Variant: If we input a time-shifted version of the input $x_1(t) = x(t - t_0)$, we get the output,

$$y_1(t) = x_1(t)\cos(\omega_c t) = x(t - t_0)\cos(\omega_c t)$$

But this is not the same as the time-shifted version of the original output, which is,

$$y(t - t_0) = x(t - t_0)\cos\big(\omega_c(t - t_0)\big)$$

Causal: Value of the output $y(t)$ does not depend on the future values of the input $x(t)$.

Memoryless: Value of the output $y(t)$ depends only on present value of the input $x(t)$. Notice ω_c is constant.

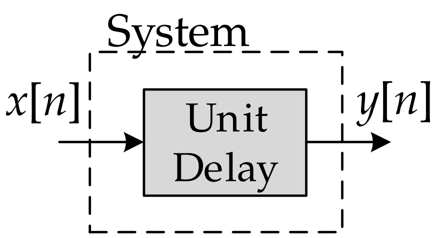
Stable (BIBO stable): Since $|\cos(\omega_c t)| \leq 1$, it follows that,

$$|y(t)| = |x(t)\cos(\omega_c t)| \leq |x(t)|$$

Hence, if the input is bounded by a value of k_1 (i.e., $|x(t)| < k_1$) the output must be bounded as well

$$|y(t)| < k_1$$

Q2. The following system has an input discrete-time signal $x[n]$ and an output discrete-time signal $y[n] = x[n-1]$. Classify this system, and clearly explain why.



Linear: If $x[n] = a\, x_1[n] + b\, x_2[n]$, then $y[n] = x[n-1] = a\, x_1[n-1] + b\, x_2[n-1] = a\, y_1[n] + b\, y_2[n]$, which is the superposition of the two outputs in response to $x_1[n]$ and $x_2[n]$ individually.

Time-Invariant: If we input a time-shifted version of the input $x_1[n] = x[n-n_0]$, we get the output,

$$y_1[n] = x_1[n-1] = x[n-1-n_0]$$

And it is the same as time-shifted version of the original output, which is,

$$y[n-n_0] = x[n-n_0-1] = x[n-1-n_0]$$

Causal: Value of the output $y[n]$ does not depend on future values of the input $x[n]$.

With Memory: Value of the output $y[n]$ depends on a past value of the input $x[n]$.

Stable (BIBO stable): If we have all bounded input samples, i.e.,

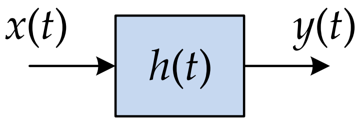
$$|x[n]| < k_1, \quad \text{for all } n$$

Then the output must be bounded as well

$$|y[n]| = |x[n-1]| < k_1$$

Unit impulse response function $h(t)$

$h(t)$ is used as a mathematical model for an LTI system. It provides the relationship between the input and output as a convolution operation.



$$y(t) = x(t) * h(t)$$

The reason for the name is that if the input signal is $x(t) = \delta(t)$ (unit impulse), then the output is $y(t) = x(t) * h(t) = \delta(t) * h(t) = h(t)$.

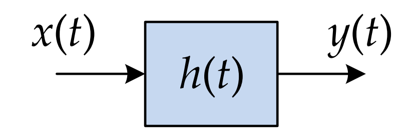
Frequency response function (or frequency transfer function) $H(\omega)$

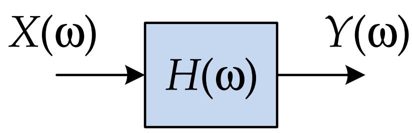
$$y(t) = x(t) * h(t)$$

$$\mathcal{F}\{y(t)\} = \mathcal{F}\{x(t) * h(t)\}$$

$$Y(\omega) = X(\omega)\, H(\omega)$$

$$H(\omega) = \mathcal{F}\{h(t)\} = \frac{Y(\omega)}{X(\omega)} = \frac{V_o(\omega)}{V_i(\omega)}$$





Hence, for many systems, it is easier to calculate the output in the frequency-domain (*multiplication*) than in time-domain (*convolution*).

$H(\omega) = \mathcal{F}\{h(t)\}$ is, in general, a complex number, so $Y(\omega) = X(\omega)\, H(\omega)$ means: $|Y(\omega)| = |X(\omega)| \cdot |H(\omega)|$ & $\angle Y(\omega) = \angle X(\omega) + \angle H(\omega)$.

A plot of $|H(\omega)|$ shows magnitude gain of the system versus ω (gain experienced by an input sinusoid at that frequency).

This $|H(\omega)|$ plot is called the **magnitude frequency response** or the **amplitude frequency response**.

A plot of $\angle H(\omega)$ shows how the system modifies or changes the phase of the input sinusoid of frequency ω.

This $\angle H(\omega)$ plot is called the **phase frequency response**.

$|H(\omega)|$ and $\angle H(\omega)$ show at a glance how the LTI system affects input sinusoids of various frequencies.